The Impact of Default Barriers on Corporate Assets

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The undersigned certify that we have read a thesis, entitled "The Impact of Default Barriers on Corporate Assets" submitted to the Graduate School by Choi Tsz Wang ( ) in partial fulfillment of the requirements for the degree of Master of Philosophy in Risk Management Science. We recommend that it be accepted.

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Declaration

No portion of the work referred to in this thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institution of learning.

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Abstract

The thesis studies the issues of estimating default barriers from market values of equities. It starts with reviewing some structural models to credit risk and estimation methods of effective parameters of those models. Then, it shows that default barriers are overstated if a widely accepted proxy for the market value of the corporate assets, the market value of equity plus the book value of corporate liabilities, is employed. In particular, this thesis applies options' properties to prove that adopting the proxy generates a positive value of default barrier no matter what market data are observed. Moreover, this "implied barrier" must be larger than the book value of corporate liabilities. To get rid of the bias, a maximum likelihood (ML) estimation approach is proposed to estimate the market values of corporate assets, the asset value volatility and the default barrier at the same time. The proposed method is used to conduct an empirical analysis with a large cross-section of industrial firms. The thesis documents that default barriers are positive but not very significant for most firms. Typically, default barriers of many firms in our sample are lower than the corresponding book values of corporate liabilities. Contrary to many people's belief, our empirical results show that the market values of firms' assets obtained by the ML approach are significantly less than those measured by the proxy.

摘要

本論文研究以股東權益市值 (market values of equities) 估計破產邊界 (default barriers) 的問題。本文首先回顧了一些信貸風險的結構性模型 (structural model) 以及其參數的估計方法，這些參數包括公司資產市值 (market value of assets) 及資產波幅 (asset volatility)。接著，本文證明了若採用股東權益市值與公司負債帳面值 (book value of corporate liabilities) 之和來代替公司資產市值，則破產邊界便會不合理地被高估。其中，我們應用期權特性來證明採用以上代替方法必迫使破產邊界的估計值大於公司負債帳面值。為了消除這種誤差，我們建議利用極大概似估計法(maximum likelihood estimation approach) 來同時估計公司資產市值、資產波幅及破產邊界。這方法被使用來進行一次包括不同工業公司的經驗分析，結果發現大部份公司的破產邊界是正值但非十分顯著，在一般情況下，許多公司的破產邊界是少於其公司負債帳面值。與大部分人所相信的不同，我們的數據結果進一步證明了從極大概似估計法中所估計的公司資產市值是遠少於從代替方法所計算出來的。

Contents

Chapter 1

Introduction

The option-theoretic approach for corporate security valuation is originated from the seminal works of Black and Scholes (1973) and Merton (1973, 1974). Based on this framework, equity value is viewed as a standard call (SC) option on corporate assets with a strike price equal to the promised payment of corporate liabilities. One recognizes that the conventional view of equity value as a SC option is inadequate to describe the consequences of bankruptcy at all times before the option's maturity. The paper of Black and Cox (1976) supplements the Black-Scholes-Merton (BSM) framework by imposing a failure barrier (default barrier) to trigger bankruptcy prior to the maturity. When the underlying asset price breaches the barrier, corporate equity can be knocked out by bankruptcy so that bond holders are able to receive the remaining value of the firm before it deteriorates further. Black and Cox (1976) stressed that the default barrier is relevant to bond protective covenants. As a result, corporate equity is modeled as a down-and-out call (DOC) option. Meanwhile, corporate debt is valued as a portfolio of default-free debt, a short put option and a long down-and-in call.

These insights have had a profound impact on financial theory as well as application. With the concept of default barrier, there are theoretical works on the debt valuation and optimal capital structure (see, for example, Longstaff and Schwartz, 1995; Leland and Toft, 1996; Brisys and de Varenne, 1997). The option-theoretic approach also facilitates the estimation of parameters of corporate bond pricing models and credit risk models. For instance, the Moody's KMV Corporation estimates values of firms' assets and volatilities by a barrier option framework where the default barrier is set as the default point, short-term debt plus half of the long-term debt, (see Crosbe and Bohn, 1993). Instead of using the subjectively defined barrier, this paper proposes a framework which objectively estimates the default barrier, the market value of the firm's assets and the asset volatility at the same time.

This thesis greatly improves the conceptual framework proposed by Brockman and Turtle (2003). In their paper, the market value of the firm's assets is assumed as the market value of equity plus the book value of corporate liabilities (the proxy[1]). Then, default barriers are extracted from the equation of setting the DOC option pricing formula to equity value. The hypothesis of positive barrier is tested statistically. Their empirical results show that implied barriers are statistically significant for a large cross-section of industrial firms. Their robustness tests reveal that default barriers remain significant over a wide range of input parameters. However, the present thesis argues that using the proxy should effectively overstate the default barrier in their estimation framework. In other words, the Brockman and Turtle (2003) framework has implicitly associated all their sample firms with positive barriers before the calibration.

This thesis applies economic theories to uncover implications of using the proxy. By properties of SC and DOC options, we prove our claim that implied barriers

[1]There are examples of using the proxy for testing structural corporate bond pricing models (see, Ogden, 1987; Lyden and Saraniti, 2000; Eom, Helwege and Huang, 2003) or for examining structures of corporate liabilities (see, Barclay and Smith, 1997a, b; Jung, Kim and Stulz, 1996).

are larger than the book value of corporate liabilities if the proxy is adopted. This claim is true no matter what empirical data are observed. The consequence is that an unbiased empirical analysis involving default barrier should not be imposed with the proxy. This motivates us to establish an alternative framework for default barrier estimation.

We propose a statistical based estimation to the DOC framework with dropping out the proxy. Like Brockman and Turtle (2003), we model the corporate equity as a DOC option on the corporate assets. In place of using the proxy, we estimate the default barrier, the market value of the firm's assets and the asset volatility by means of maximum likelihood (ML) estimation. Previous works on the ML estimation concentrate on estimating firm's assets and volatility (see, for example, Duan, 1994; Ericsson and Reneby, 2002). Our approach extends the ML estimation to include the barrier estimate with the help of the mathematical formula of the DOC option. As ML estimation is a well-developed statistical concept, parameter values obtained from the method can be used to access the quality of the proxy and its influence to the barrier estimate.

This thesis contributes to the literature by theoretically recognizing the implication of using the proxy, by empirically examining the significance of default barriers and by proposing an objective framework to capture barriers' effect on corporate claims. We estimate the default barriers, the market values of firms' assets and the asset volatilities for a sample of 13,317 firm-years. Contrary to many people's belief, we find that the estimated value of firms' assets is significantly smaller than those approximated by the proxy. The thesis shows that most estimated barriers, i.e. over 70% of our sample, are positive but less than the book value of corporate liabilities. We also find that some industrial sectors contain a group of companies with zero barriers as well as a group of companies with barrier levels over the corporate liabilities. This indicates that companies in the same sector may not share a common default structure in terms of default barriers. Hence, parameter estimation is better performed in a firm-specific manner.

The rest of the thesis is organized as follows. Chapter 2 reviews the theoretical structural models of Merton (1974) and Black and Cox (1976), the cornerstone of corporate security valuation. Chapter 3 examines two estimation methods for parameters of corporate security valuation. Chapter 4 derives implications of using the proxy by financial arguments. Chapter 5 proposes the ML estimation method. We give a simulation-based verification to the proposed approach. Empirical results, together with economic interpretations, are reported in Chapter 6. A conclusive remark is made in the final chapter.

Chapter 2

Review of Structural Models

2.1 The Merton model

Regarding the past literature, rating agencies models, reduced-form models and structural models are the three main types of models to deal with credit risks. Rating agencies models are a class of discrete time model which attempt to present the default event or change of credibility in terms of probability. It cannot cope with continuous time financial situation. Reduced-form models treat default as an unpredictable event governed by a hazard rate process but the estimation method for the hazard process is not well developed so far. In this thesis, our approach towards credit risks would be based on structural models. This is because structural models can manage continuous time financial dynamics and estimation methods for effective parameters in these models are numerous.

In Merton's (1974) model, suppose the capital structure of a firm includes both equity and debt, the equity-holders are residual claimants on the firm's assets after all obligations have been met. The value of equity would be the firm assets value less the debt if the firm is able to pay its obligations. Otherwise, it would be equal to zero when the firm fails to fulfill its liabilities. Therefore, the equity of a firm can be viewed as a call option on the firm's assets. The strike price of the call option is the book value of firm's liabilities. The market value of the underlying assets follows a Geometric Brownian Motion (GBM):

$$dV = \mu V dt + \sigma V dW, \tag{2.1}$$

where V is the firm's assets value, with an instantaneous drift μ and an instantaneous volatility σ. W is a standard Wiener process. By the Itô Lemma, the dynamic of the value of firm's assets can be rewritten as

$$d \ln V_t = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma \, dW_t \tag{2.2}$$

and the value of firm assets at any time t can be given by

$$\ln(V_{t+T}) = \ln(V_t) + \left(\mu - \frac{\sigma^2}{2}\right) T + \sigma\sqrt{T}\epsilon_{t+T} \tag{2.3}$$

where ϵ_{t+T} follows standard normal distribution.

We denote by X the book value of the debt which has a maturity of T. Then, the Merton model says that the market value of equity, V_E, relates to V and X by the Black-Scholes formula:

$$V_E = VN(d_1) - Xe^{-rT}N(d_2) \tag{2.4}$$

where

$$d_1 = \frac{\ln(V/X) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \; d_2 = d_1 - \sigma\sqrt{T}$$

and r is the risk-free rate and $N(\cdot)$ is the cumulative density function of a standard normal random variable.

The Merton model enables us to derive default probability of a firm. It is the probability that the firm's assets will be less than the book value of firm's liabilities.

In other words,

$$DP_T = Pr(V_{t+T} \leq X_t | V_t) = Pr(\ln(V_{t+T}) \leq \ln(X_t) | V_t) \tag{2.5}$$

Substituting (2.3) into (2.5), we can express the default probability as follows:

$$DP_T = Pr\left(\ln(V_t) - \ln(X_t) + \left(\mu - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}\epsilon_{t+T} \leq 0\right)$$

$$DP_T = Pr\left(\epsilon_{t+T} \leq -\frac{\ln\left(\frac{V_t}{X_t}\right) + \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right) \tag{2.6}$$

We can then define the distance to default (DD) as follows:

$$DD_t = \frac{\ln\left(\frac{V_t}{X_t}\right) + \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{2.7}$$

Default occurs when the ratio of the value of assets to debt (quasi-debt ratio) is less than one, or its log is negative. The DD measures the number of standard derivations that the log-quasi-debt ratio derivates from its mean before default happens. Notice that although the value of the call option in (2.4) does not depends on μ, DD does. This is because DD depends on the future value of assets which is given in (2.3). We use the standard normal distribution implied by Merton's model. In this case, the theoretical probability of default will be given by

$$DP_T = N(-DD) = N\left(-\frac{ln\left(\frac{V_t}{X_t}\right) + \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right) \tag{2.8}$$

which illustrates how Merton model calculates default risk, see Crosbe and Bohn (1993).

2.2 The default barrier model of Black and Cox

Based on the idea of Merton (1974), Black and Cox (1976) improves the security valuation of standard call option by considering the effects of safety covenants. In fact, the Merton model assumes that the bond contract would place limiting restrictions on firm assets value. These restrictions may impose both lower and upper boundaries at which the asset value must take on specific values. The boundaries may be given exogenously by contact specifications or determined endogenously as an optimal decision problem.

Black and Cox (1976) figures out the impact of safety covenants on the value and behavior of the firm's assets. Safety covenants are contractual provisions which give the bondholders the right to bankrupt or force a reorganization of the firm if it is doing poorly according to some standards. These standards induce the realization of default barrier for the value of firm's assets. A natural form of a safety covenant is the following: if the value of the firm falls to a specified level, H, which may change over time, then the bondholders are entitled to force the firm into bankruptcy and obtain the ownership of the assets. Under this assumption, the distribution of the value of the firm assets at time τ, V_τ, in a risk neutral world conditional on its value at the current time t, V_t, will follow lognormal distribution with an absorbing barrier. Using the idea of Merton that the equity value can be regarded as a call option on the firm asset value, the present problem is equivalent to solve the Black-Scholes Equation

$$\frac{\partial V_E}{\partial t} + \frac{1}{2}\sigma^2 V^2 \frac{\partial^2 V_E}{\partial V^2} + rV\frac{\partial V_E}{\partial V} - rV_E = 0$$

with boundary conditions

$$V_E(T, V_T, H) = \max(V_T - X, 0)$$

$$V_E(t, H, H) = R$$

where $V_E(t, V_t, H)$ is the value of equity at time t with firm asset value V_t and default barrier H and R is the residual claims given to the equity holders.

The time of beaching the barrier would be a random variable. It distributes as the first passage time to the barrier, and the approach taken by Cox and Ross (1975) can be applied. As a result, the value of a firm's equities would be changed from a standard call (SC) option of (2.4) to a Down-and-Out Call (DOC) option with the following pricing formula:

$$
\begin{aligned}
V_E &= \mathbf{DOC}(V, X, H) \\
&= VN(a) - Xe^{-rT}N\left(a - \sigma\sqrt{T}\right) \\
&\quad -V(H/V)^{2\eta}N(b) + Xe^{-rT}(H/V)^{2\eta-2}N\left(b - \sigma\sqrt{T}\right) \\
&\quad +R(H/V)^{2\eta-1}N(c) + R(V/H)N\left(c - 2\eta\sigma\sqrt{T}\right),
\end{aligned} \tag{2.9}
$$

where

$$
\begin{aligned}
a &= \begin{cases} \frac{\ln(V/X)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}, & \text{for } X \geq H, \\ \frac{\ln(V/H)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}, & \text{for } X < H, \end{cases} \\
b &= \begin{cases} \frac{\ln(H^2/VX)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}, & \text{for } X \geq H, \\ \frac{\ln(H/V)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}, & \text{for } X < H, \end{cases} \\
c &= \frac{\ln(H/V) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \quad \text{and} \quad \eta = \frac{r}{\sigma^2} + \frac{1}{2}
\end{aligned} \tag{2.10}
$$

and R is the rebate paid to the equity-holders if the firm assets value breaches the barrier.

Chapter 3

Estimating the Merton model

3.1 The Variance Restriction (VR) method

Based on the Merton (1973) model, Ronn and Verma (1986) proposes a methodology, which is termed Variance Restriction (VR) method, for arriving estimates of deposit insurance premiums from market data by using isomorphic relationships between equity and a call option, and insurance and a put option. It derives the fair price of the deposit insurance by the argument of Merton (1973) as a standard put option on the market value of the bank. The maturity of this put option would be the length of time until the next audit of the bank's assets by the insurer. If we make the standard assumptions of Black-Scholes option pricing model, then the analytic representation of the *per dollar* deposit insurance premium, denoted d, would be

$$d = N(y + \sigma\sqrt{T}) - (V/X)N(y) \tag{3.1}$$

where

$$y = \frac{\ln[X/V] - \sigma^2 T/2}{\sigma\sqrt{T}}$$

and V is the unobserved value of the bank's assets, X is the face value of total liabilities, σ is the instantaneous standard deviation of the rate of return of the bank's assets, T is the time until next audit of the bank's assets and $N(\cdot)$ is the cumulative density of a standard normal random variable.

The chief obstacle to the empirical application of the model lies in the fact that neither the true value of the firm, V, nor its instantaneous volatility, σ, can be empirically observed. This problem can be resolved by making use of the property that the equity of a firm can be regarded as a standard call option on the value of firm assets. The idea has been introduced in Chapter 2. Thus, as we let V_E as the equity of the bank,

$$V_E = VN(d_1) - XN(d_1 - \sigma\sqrt{T}) \tag{3.2}$$

where

$$d_1 = \frac{\ln(V/X) + \sigma^2 T/2}{\sigma\sqrt{T}}$$

V_E can be rewritten as $V_E(t, V)$ since it can be viewed as a call option which depends on current time and bank asset value. By assuming V follows lognormal distribution (2.1) and applying the Itô Lemma, we have

$$dV_E = (\cdot)dt + \frac{\partial V_E}{\partial V}\sigma V dW \tag{3.3}$$

Dividing the both sides of (3.3) by V_E and taking variance, it yields

$$\sigma_E = \frac{V(\frac{\partial V_E}{\partial V})}{V_E}\sigma \tag{3.4}$$

where σ_E is the instantaneous standard derivation of the return on V_E.

Equation (3.2) and (3.4) can be solved simultaneously for two unknowns, V and σ, by a numerical routine for each observed V_E and σ_E (where the latter is estimated from daily return time series for the concerned period). Then, given the solution pair (V, σ), an estimate of the deposit insurance premium can be computed using equation (3.1).

3.2 The Maximum Likelihood (ML) method

The VR methodology for computing firm assets value and its volatility has been criticized by Duan (1994), which pointes out that under the model specification of Ronn and Verma, the asset price follows a lognormal process and equity is viewed as a call option on the bank assets. This implies that the equity volatility must be stochastic. In fact, the volatility relationship (3.4) is a redundant condition which provides a restriction only because the equity volatility is inappropriately treated as a constant. Duan (1994) calculates the deposit insurance premium through utilizing statistical maximum likelihood techniques for estimating the two unknowns.

As the equity value of the bank can be viewed as a call option on the bank assets, the Black-Scholes option pricing formula (2.4) thus defines a one-to-one mapping between the unknown asset value, V^t, and the observed equity value, V_E^t, at every time t. Together with the lognormal assumption of the bank assets, one can use standard theory on differentiable transformation to derive the log-likelihood function for the observed sample of equity values. That is,

$$\begin{aligned} L(V_E^t, t = 1, \cdots, n; \mu, \sigma) &= -\frac{n-1}{2}\ln(2\pi) - \frac{n-1}{2}\ln(\sigma^2) \\ &\quad - \sum_{t=2}^{n} \ln(\hat{V}^t(\sigma)) - \sum_{t=2}^{n} \ln(N(\hat{d}_1^t)) \\ &\quad - \frac{1}{2\sigma^2}\sum_{t=2}^{n}\left[\ln\left(\frac{\hat{V}^t(\sigma)}{V^{\hat{t}-1}(\sigma)}\right) - \mu\right]^2 \end{aligned} \tag{3.5}$$

where $\hat{V}^t(\sigma)$ is the unique solution to (2.4) for any σ, and $\hat{d}_1^t$ corresponds to d_1 with $\hat{V}^t(\sigma)$ in place of V^t. With the log-likelihood function in (3.5), iterative optimization routine can be employed to compute the maximum likelihood estimates $\hat{\mu}$ and $\hat{\sigma}$. The deposit insurance premium can then be obtained by substituting $\hat{\sigma}$ and $\hat{V^t}(\sigma)$ in (3.1).

3.3 Comparison between VR and ML methods

Except from calculating deposit insurance premium, the mentioned methodologies are also used to implement structural bond pricing models. Ericsson and Reneby (2002) performs a simulation experiment in order to evaluate the maximum likelihood (ML) method (proposed in Duan (1994)) and the variance restriction (VR) method (proposed in Ronn and Verma (1986)) to this application. The properties of the bond price estimators are examined through four theoretical bond pricing models: the Black & Scholes (1973) / Merton (1974) model, the Leland and Toft (1996) model, the Byris and de Varenne (1997) model, as well as the Ericsson and Reneby (2001) model.

Ericsson and Reneby first simulates an asset value path according to the respective dynamics of models. Second, it computes the corresponding stock price path utilizing the relevant equity value formula. This equity path would be acted as the input of ML and VR approach to solve for the asset value and its volatility. Finally, it uses the estimates $(\hat{V},\ \hat{\sigma})$ to calculate the bond prices, the mean error and standard deviation of the estimates. These steps are repeated for each sample path in order to assess the sampling distribution for all models.

From the summary across all models, the results are strongly supportive to the performance of ML estimators. With true current asset value of 1000, the mean error(and standard derivation) of ML estimators is 1(15) while that of VR estimators is -22(61) . To asset volatility, the mean error(and standard derivation) is

-0.1%(2.0%) and 6.7%(16.6%) for ML and VR estimators respectively. The bias of the ML approach in bond pricing is negligible for practical purposes. This result holds for all models. In contrast, the VR approach exhibits an average pricing error of -4.5% and the bias varies among different models. In sum, this simulation experiment suggests that the ML approach would be a better choice for estimating the unobserved asset value and its volatility since it renders a higher accuracy. In addition, only two parameters can be estimated from the VR approach. This restriction is imposed by solving a system of two equations in the approach. However, there is no such limitation for the ML approach. More parameters can be estimated given that we are able to derive the required likelihood function.

Chapter 4

Implications of Using the Proxy in Default Barrier Estimation

In Chapter 3, VR and ML methodologies have been presented to find the market value of firm's assets and asset volatility. One of its possible weaknesses is that both methodologies cannot respect the effect of safety covenants which has been mentioned in Chapter 2. In order to predict the influence of safety covenants on the market value of firm's assets, different approaches are proposed to estimate the default barrier.

A typical example of default barrier estimation with the proxy refers to the recent paper of Brockman and Turtle (2003). Their paper starts by viewing the market value of equity, V_E, as a DOC option on the market value of corporate assets, V, with an exercise price equal to the future promised payment (all non-equity liabilities), X, with provision on a constant barrier level, H. The market value of equity and the future promised payment are obtained from the market information and the accounting information of firms. The market value of the corporate assets is measured as the proxy $(\tilde{V} = V_E + X)$. The asset volatility is measured as the annualized standard deviation of the proxy. With assuming zero rebate, barriers are calibrated by setting the DOC option pricing formula to the market value of equity. Since the DOC price collapses to the SC price when the barrier level is zero in value, their paper argues that the positive barrier hypothesis is testable. Their statistical tests report that implied barriers are significantly positive with over 99% confidence.

By looking at their results carefully, we discover a very interesting phenomenon. In their sample, most (if not all) implied barriers are greater than the future promised payment of corporate liabilities. This observation can be deduced from the Panel D of Table 2 of their paper. In this chapter, we show that all findings mentioned above are the consequences of using the proxy.

4.1 Rejection of SC framework

We now focus on the implication of using the proxy, $\tilde{V}$, under the SC framework to security valuation first. By regarding the corporate equity as the subject, the definition of the proxy can be re-written as

$$V_E = \tilde{V} - X,$$

where the right hand side exhibits the intrinsic value of the SC option written on the proxy, $\tilde{V}$, with a strike price equal to the value of liabilities, X. Using the no arbitrage pricing principle, standard textbooks on options, like Hull (2001) and others, derive a model-independent result that

$$\tilde{V} - X < \tilde{V} - Xe^{-rT} \leq \mathbf{SC}(\tilde{V}, X).$$

where $\mathbf{SC}(\tilde{V}, X)$ is the pricing formula for SC options. The inequality implies that the SC framework to security valuation is rejected if the proxy is used.

4.2 Positive barrier implication

In fact, the proxy forces implied barriers to be positive in the DOC framework. The pricing formula for DOC option has a property that the option premium can be adjusted to any value between the rebate, R, and the SC price by altering the barrier level, H. Specifically, the DOC price decreases from the SC price to the rebate value by increasing the barrier level, H, from 0 to the asset value. With a rebate that is no greater than the intrinsic value, a positive H must be chosen such that the DOC price is equal to the intrinsic value. In other words, the DOC property shows that a positive barrier is generated as a result of employing the proxy. It is important to notice that this implication is true for any set of input parameters since it is derived from the model-independent properties of DOC options. Examples of input parameters include industrial sector, value of the firm's liabilities, asset volatility, option maturity and rebate level. The implication tells us a fact that the hypothesis test and robustness tests of Brockman and Turtle (2003) work extra-ordinarily well because all their sample firms have been associated with positive barriers, since the proxy is employed.

4.3 Barrier over debt implication

Nonetheless, adopting the proxy presumes that the default barrier level is greater than the value of the firm's liabilities. To see this, we denote $\mathbf{DOC}(V, X, H)$ as the current price for a DOC option on V with a strike price of X and a barrier of H. By the no arbitrage pricing principle, we argue that

$$\mathbf{DOC}(V, X, X) > V - X.$$

If it is not the case(i.e. $V - \mathbf{DOC}(V, X, X) - X \geq 0$), an investor can make an arbitrage profit by selling the asset at V to purchase the DOC option. The remaining cash are put into a bank account. Then, the investor can take different actions for two possible scenarios:

1. The asset price V does not breach the barrier level X prior to the maturity. On the maturity day (T), she will exercise the option to purchase the asset by a value of X so that her short position in the asset can be cancelled. Then, an arbitrage profit of

$$[V - \mathbf{DOC}(V, X, X)]\, e^{rT} - X$$

is made at time T.

2. In case the asset value breaches the barrier level X at time $\tau < T$, the investor will receive a rebate of R. She will purchase the asset right away with an amount of X to cancel her short position in the asset. Then, an arbitrage profit of

$$[V - \mathbf{DOC}(V, X, X)]\, e^{r\tau} - X + R$$

is made at time τ.

As a result, the no arbitrage price of DOC options should satisfy the preceding inequality. This inequality implies that a DOC option price equals its intrinsic value only when the barrier level H is strictly greater than the strike X. Mathematically, we write:

$$\begin{aligned} V - X = \mathbf{DOC}(V, X, H) \;&\Rightarrow\; \mathbf{DOC}(V, X, X) > \mathbf{DOC}(V, X, H) \\ &\Rightarrow\; H > X. \end{aligned} \tag{4.1}$$

The last line of (4.1) is true because the DOC pricing formula is decreasing with the barrier level.

In the paper of Brockman and Turtle (2003), the DOC option pricing formula, which is derived from the no arbitrage argument, and the proxy is used at the same time. As mentioned, this automatically assigns a DOC option price equal to its intrinsic value. According to the last implication (4.1), the "implied barrier" obtained from their DOC framework must excess the book value of corporate liabilities. This gives a hint to the observation mentioned in the early part of this chapter.

The barrier-excess-liability feature is quite unusual, especially when the rebate paid to equity holders is zero. When the underlying asset price breaches the barrier, which is strictly greater than the liability, bond holders are able to get their loans back but the equity holders receive nothing, due to the zero rebate assumption. It is so strange that the remaining asset value, after paying loans, evaporates. Therefore, the DOC framework of Brockman and Turtle is a self-conflicting approach.

4.4 Numerical illustration

In order to further illustrate our arguments, a numerical example is constructed by solving H from the equation,

$$V - X = \mathbf{DOC}(V, X, H), \tag{4.2}$$

with various input parameters, where the mathematical formula for the DOC option is presented in (2.9) of Chapter 2. We tabulate our results together with that of the robustness tests of Brockman and Turtle (2003), in Table 4.1, for comparative purpose. To match with their scale, we use the market value of corporate assets of 1.0, the future promised payment of 0.45, the risk-free rate of 5% and the base asset volatility of 25% in our computation. It is important to notice that our computation involves no empirical data.

Table 4.1
A comparison to the results of Brockman and Turtle (2003)

Panel A: barrier estimates for various option lives with fixed volatility and zero rebate

	3 Years	5 Years	10 Years	30 Years	100 Years
Solving (4.2)	0.6543	0.6623	0.6839	0.7208	0.7352
Brockman and Turtle	0.6772	0.6802	0.6920	0.7137	0.7224

Panel B: barrier estimates for rebates of 0 , 5, 10, 15 and 20%.

	0 %	5 %	10 %	15 %	20 %
Solving (4.2)	0.6839	0.7067	0.7307	0.7560	0.7825
Brockman and Turtle	0.6920	0.7123	0.7334	0.7553	0.7777

Panel C: barrier estimates for volatilities of 80 , 90, 100, 110 and 120% of the base case volatility[a]

	80 %	90 %	100 %	110 %	120 %
Solving (4.2)	0.7377	0.7091	0.6839	0.6619	0.6425
Brockman and Turtle	0.6991	0.6954	0.6920	0.6884	0.6844

[a]We use the fixed value of 0.25 as the base case volatility whereas the paper of Brockman and Turtle (2003) uses volatilities derived from the data, which vary for different observations.

The results in Panel A and Panel B of Table 4.1 show that barrier levels obtained by solving (4.2) are very close to the averaged normalized barrier levels obtained in Brockman and Turtle (2003) in terms of both the order of magnitude and the increasing trends. Panel C of Table 4.1 reveals that the decreasing trend of the barrier levels obtained by the two methods are coherent with each other. Barriers implied by the two methods have a slightly difference in values because the base volatility used in Brockman and Turtle (2003) is the volatility of the proxy firm values whereas we use a fixed value of 25%. All barriers implied from the two methods are larger than the liability level of 0.45. Thus, the phenomena of the barrier levels obtained by two methods agree with each other exactly.

The numerical illustration conveys two important messages. First, it shows that we understand Brockman and Turtle framework in a correct way. This claim can be proved by the facts that barrier levels obtained from two methods are very close in terms of both the order of magnitude and trends. Also, the numerical illustration reveals that the empirical study of Brockman and Turtle (2003) does not respect any empirical data. We can randomly pick some numbers for parameter values and solve H by (4.2). Then, we can obtain similar results and draw the same interpretation as Brockman and Turtle.

Chapter 5

The Proposed Framework

In this chapter, an alternative framework for estimating default barriers is proposed and justified carefully. To be objective, we give up the proxy so that the number of estimating variables increases from one, the barrier level only, to three, including the barrier level, the market value of corporate assets and the asset volatility, at each time point. Therefore, the new framework should be able to manage the increase of estimating variables as well as to maintain the performance of estimation.

The proposed framework starts from viewing the equity value as a DOC option on the corporate assets. We assume that the underlying asset price evolves as the Black-Scholes dynamics. Specifically, in a risk-neutral world, the process for the log-asset-value, $w_t = \ln V_t$, takes the form,

$$dw_t = \left(r - \sigma^2/2\right) dt + \sigma dZ_t, \tag{5.1}$$

where V_t is the market value of the firm's assets at time t, σ is the asset value volatility, r is the risk-free interest rate and Z_t is a Wiener process. Equation (5.1) enables us to derive the closed form solution for DOC options. In the present case, the market value of equity, V_E, is given by (2.9) and (2.10). Notice that the SC option framework is incorporated by setting H to zero. This idea agrees with

Brockman and Turtle (2003).

5.1 Maximum likelihood estimation

Given a time series of market values of equities, say $\{V_E(t_i)|i = 1, 2, \cdots, n\}$, we propose to estimate the asset value volatility σ, barrier level H and a series of market values of corporate assets $\{V(t_i)|i = 1, 2, \cdots, n\}$ through ML estimation.

The idea of this proposed framework is originated from the paper of Duan (1994) which establishes a ML approach for estimating volatility and spot prices from SC options. Recently, Ericsson and Reneby (2002) compares the performances of ML estimation and the traditional method of Ronn and Verma (1986) in the context of estimating volatility and spot prices by simulation. It shows that the accuracy of the ML approach dominates the traditional ones although the latter may require less computing time. Details can be referred to Chapter 3.

The ML approach is adopted not only because of its accuracy but also the limitation of the traditional approach, which is specifically designed to estimate the volatility and the spot prices. In fact, the traditional approach involves solving the two parameters from a system of two equations. They are the price matching equation and the volatility matching equation, respectively. There is no room for us to capture the third parameter, default barrier, from the two equations as well. In contrast, the ML estimation would not induce the same problem since maximization is concerned. To our understanding, this thesis is the first work attempting to estimate the barrier level through a ML approach.

The mechanism of the proposed ML approach is indeed very simple. We denote $f(V_E(t_i)|V_E(t_{i-1}), \theta)$ as the probability density function for the equity value at time t_i conditional on the equity value at time t_{i-1} and a parameter vector θ. The ML approach estimates the value of θ such that the log-likelihood function,

$$L(\theta)=\sum_{i=2}^{n}\ln f\left(V_E^i|V_E^{i-1},\theta\right),\quad V_E^i\equiv V_E(t_i)$$

is maximized. If the density function can be expressed in closed form, then the maximization problem becomes relatively simple.

Fortunately, the conditional density function, $f(\cdot|\cdot)$, for equity value can be derived from the DOC formula of (2.9). We denote $g(w_i|w_{i-1},\theta)$ as the density function of w_i conditional on the values of w_{i-1} and θ, where w_i is the log-asset-value at time t_i. With the help of (2.9), standard change of variable technique is applied to obtain

$$f(V_E^i|V_E^{i-1},\theta)=\left[g(w_i|w_{i-1},\theta)\times\left(\frac{\partial V_E}{\partial w}\right)^{-1}\right]_{w_i=w\left(V_E^i,t_i;\underline{\theta}\right)},\tag{5.2}$$

where $\underline{\theta}\subset\theta$ is the subset of the parameter vector, which is necessary for pricing equity values. In fact, w_i is obtained inversely from (2.9) with the values of V_E^i and $\underline{\theta}=(\sigma,H)$ given. The partial derivative appeared in (5.2) can be implemented through the delta of DOC options as below:

$$\left.\frac{\partial V_E}{\partial w}\right|_{w=w_i}=V_i\left.\frac{\partial V_E}{\partial V}\right|_{V=V_i}=V_i\Delta(V_i,\theta).$$

As the barrier option model is concerned, the conditional density function, $g(\cdot|\cdot)$, for the random variable w_i should incorporate the feature of having an absorbing boundary to ensure that market value of the firm's assets would not go under the barrier in between any two successive time points. This density function is available in the literature (see, for example, Rubinstein and Reiner, 1991) that

$$g(w_i|w_{i-1};\theta)\ =\ \varphi(w_i-w_{i-1})-e^{2\eta(h-w_{i-1})}\varphi(w_i+w_{i-1}-2h),\tag{5.3}$$

where

$$\begin{aligned}\delta t_i &= t_i - t_{i-1}, \quad h = \ln(H),\\ \varphi(x) &= \frac{1}{\sigma\sqrt{2\pi\delta t_i}} \exp\left\{-\frac{[x-(r-\sigma^2/2)\delta t_i]^2}{2\sigma^2\delta t_i}\right\},\end{aligned} \tag{5.4}$$

and η is defined in (2.10). It is worth to mention that the function $g(\cdot|\cdot)$ takes the form as (5.3) if the underlying asset value is larger than the barrier. Otherwise, its value is set to zero. As a result, the survivorship of firm is actually taken into account when computing the log-likelihood function. The survivorship consideration is also discussed in Duan et al. (2003).[1]

Let us summarize the whole estimation procedure. After specifying the log-likelihood function as

$$L(\theta) = \sum_{i=2}^{n} \left[\ln g(w_i|w_{i-1}) - \ln\left[V_i\Delta(V_i;\underline{\theta})\right]\right]_{w_i=w(V_E^i,t_i;\underline{\theta})}, \tag{5.5}$$

the estimates of asset value volatility, $\widehat{\sigma}$, and the barrier level, $\widehat{H}$, are obtained by maximizing equation (5.5). Finally, a time series of estimated market value of the firm's assets are obtained from the inverse of the equity pricing function :

$$\widehat{w_i} = DOC^{-1}\left(V_E^i, t_i; \widehat{\sigma}, \widehat{H}\right).$$

5.2 Barrier-to-debt ratio specification

When handling with real data, we find that the liability of a firm, X, can change year by year so that it is not desirable to have a constant barrier level across years. At the same time, the ML approach requires a reasonably large sample size to keep

[1] We recognize the paper of Duan et al.(2003) after completing this paper. Our work is different from theirs in two major ways. First, we allow the firm to default at any time, once the asset value breaches the barrier. Whereas their paper considers defaults happen at discrete re-financing moments. Second, our framework includes the estimation of the default barrier but they concentrates on estimating the drift of the business.

the estimation standard. That means cross-year data are better used. To strike the balance for carrying on the estimation, we make the assumption that the barrier level is proportional to the liability by a constant value. Specifically,

$$H = \alpha X, \tag{5.6}$$

with α being a constant. Therefore, the proposed framework estimates the value of α instead of that of H.

This assumption allows us to draw economic interpretation for the barrier level as well as not affecting the original purpose of validating non-zero implied barriers. Once the barrier level is indeed zero in value, then equivalently a zero α will be obtained. As a byproduct, the value of α gives us an idea where the barrier level should be. When α is less than one, the barrier level is located under the liability. In this case, the amount of liabilities should be concerned if the bankruptcy risk is concerned. Otherwise, the default barrier dominates the liability level in triggering the bankruptcy process.

5.3 Simulation checks

A Monte Carlo simulation is designed to check the reliability of our maximum likelihood estimation approach. This is significant in two folds. First, as the continuous time model is discretized to match with the real data, it is vital to know if any bias occurs in parameter estimates. Second, estimating the barriers by maximum likelihood approach is not studied before. It is essential to examine the approach to see its performance.

The simulation involves generating 30 sets of market value of the firm's assets according to the dynamics of (5.1). Each of which comprises of 2,600 equally timespacing asset values, replicating 10 years daily observations, with initial values of 1, i.e. $V_0 = 1$. For each generated time series, three sets of market value of equity are produced via the DOC option pricing formula (2.9) for $\alpha = 0.5, 1$ and 1.5 with other parameter values being fixed. Specifically, we adopt $r = 0.05$, $T = 10$, $\sigma = 0.25$ and $R = 0$ in (2.9). The future promised payment is allowed to change with time that varies from 0.36 to 0.45 linearly. After that, 30 time series of market values of equities are obtained for each α value through the formula of (2.9). These 30 time series of equity values replicate the market information of an industrial sector with 30 companies. In our sample, most industrial sectors contain approximately or more than 30 companies.

With viewing every set of equity values as real data observed in the market, the estimation process runs as what we have mentioned. Table 5.1 summarizes the results.

Table 5.1
Results of Monte Carlo simulation check

Panel A: α estimates			
True value of α:	0.5	1	1.5
mean:	0.3725	0.8022	1.4763
standard derivation:	0.3278	0.3652	0.2202
Panel B: Firm asset volatility[b]			
True value of α:	0.5	1	1.5
mean:	0.2491	0.2503	0.2543
standard derivation:	0.0042	0.0065	0.0157
Panel C: Percentage error of firm asset value			
True value of α:	0.5	1	1.5
mean:	0.0011	0.0105	0.0322
standard derivation:	0.0017	0.0278	0.1297

[b]The true volatility is 0.25

The Panel A of Table 5.1 reports the α estimates by the ML approach. It shows that the accuracy of α estimates is higher when the true α is higher than 1. The average α estimates with true α value of 0.5 and 1 and 1.5 are 0.3725, 0.8022 and 1.4763 respectively. The α in the first 2 cases are underestimated for about 20% while it is very close to the true value in the last case. The standard derivation of estimates decreases with the increase of true α.

Panel B shows that the average firm asset value volatility estimates perform equally well with various true α. All estimates are very close to the true value of σ which is 0.25. Also, the standard derivation of volatility estimates keeps small and below 0.02 even if the α is different.

In Panel C, we calculate the percentage difference between the firm asset values from original time series and that from simulations at each time point. The percentage difference of a simulation is obtain by

$$\begin{array}{c}\text{Percentage difference}\\ \text{of firm asset values}\\ \text{in one simulation}\end{array} = \frac{1}{2600}\sum_{i=1}^{2600}\left|\frac{V^i_{original} - V^i_{ML}}{V^i_{original}}\right| \tag{5.7}$$

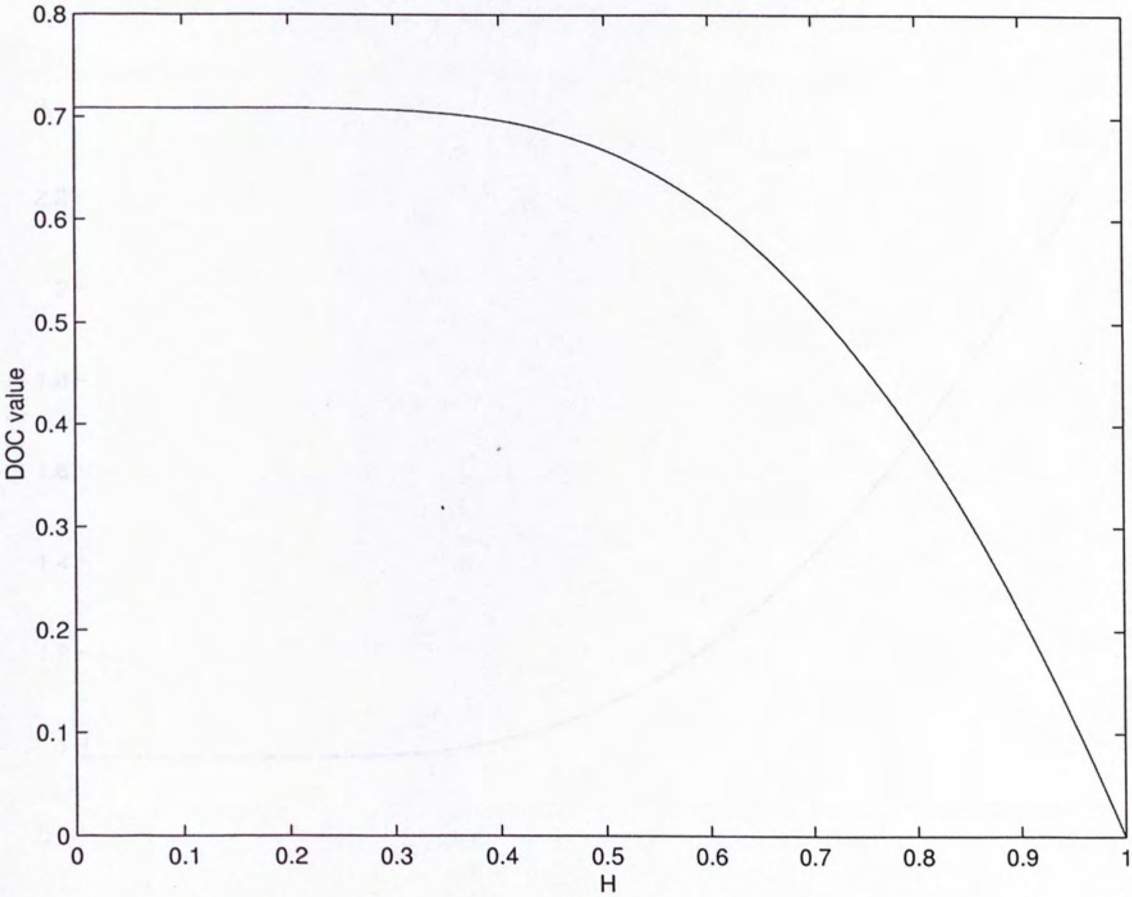
The averaged percentage difference of all simulations are 0.1%, 1% and 3.2% for true α of 0.5, 1 and 1.5 respectively with low standard derivation of around 0.0002 to 0.005 for all three cases.

The simulation verifies that our proposed framework would render a precise estimation of barrier-to-debt ratio, α, firm asset value, V, and asset volatility, σ, if the default barrier is over the liability level. It becomes crucial for corporate security valuation once the default barrier is really dominating the liability in the bankruptcy analysis. The satisfaction in estimating parameter values would give us a merit when accessing creditworthiness of companies. For the true value of α not greater than one, the ML approach underestimates the α but maintains excellent estimation for firm asset values and volatilities. This would be an advantage for us to work out the error in estimating firm asset values with the proxy.

5.4 Comments on the performance of α

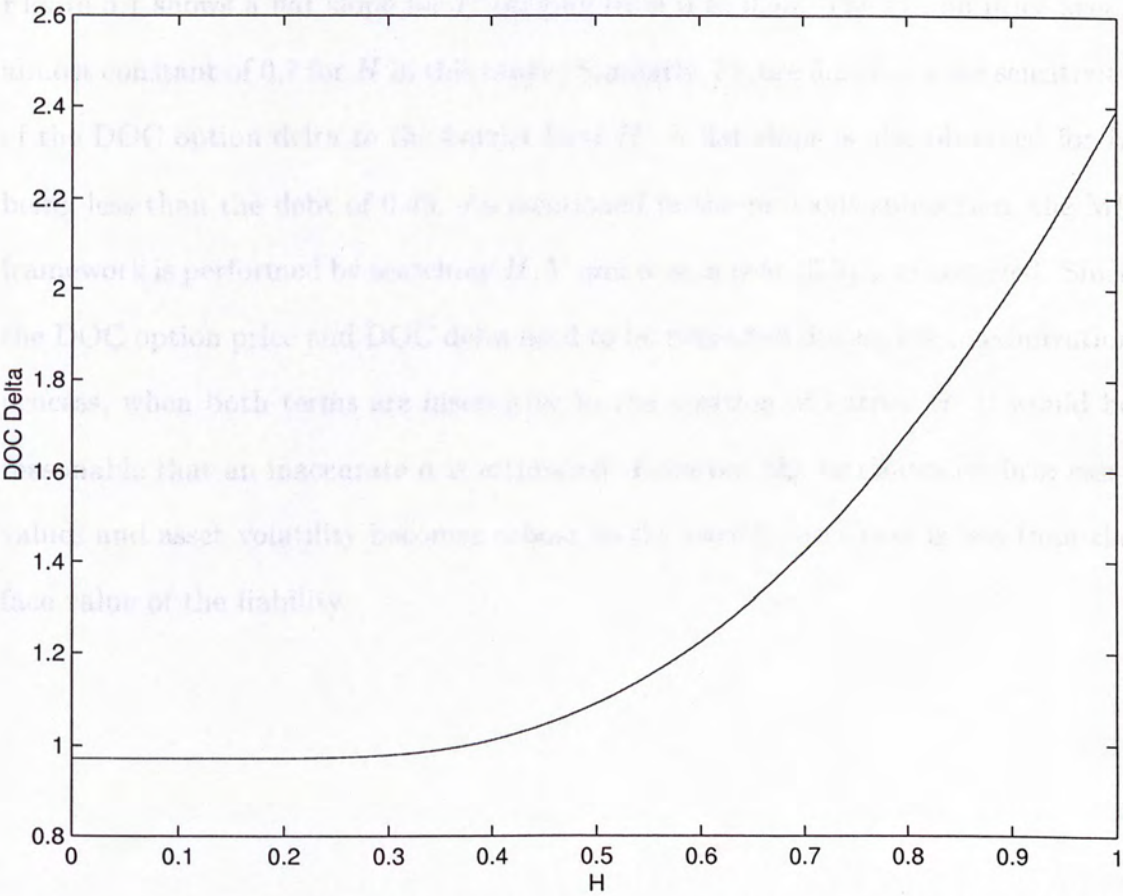
Why the α is underestimated for true values being 0.5 and 1 but it is fine for that being 1.5? It can be explained by the sensitivity of the DOC price and delta to the barrier level. When the barrier is higher than the strike, the option price and delta are sensitive enough to make the log-likelihood function capture the barrier position. Whereas the log-likelihood function would be unable to detect the barrier well since the change in barrier level does not lead to significant change in the log-likelihood function for the barrier lower than the debt. Fortunately, this does not affect the estimation standard for other parameters, like firm asset values and asset volatility.

Figure 5.1: *The DOC option value vs Barrier.*



Parameter values used are: $V = 1, X = 0.45, r = 0.05, \sigma = 0.25$ and $T = 10$. The DOC option values are computed by varying H from 0 to 1. The slope is almost flat when H is between 0 to 0.45 and it decreases afterwards. This indicates the insensitivity of option price to default barrier when the debt-to-liability ratio (α) is smaller than or equal to 1.

Figure 5.2: *The DOC delta vs Barrier.*



Parameter values used are: $V = 1, X = 0.45, r = 0.05, \sigma = 0.25$ and $T = 10$. The DOC option deltas are computed by varying H from 0 to 1. The slope is almost flat when H is between 0 to 0.45 and it increases afterwards. This indicates the insensitivity of option delta to default barrier when the debt-to-liability ratio (α) is smaller than or equal to 1.

Figure 5.1 and 5.2 are given to illustrate the idea. In the two figures, we employ the following set of parameter values: the firm asset value V of 1.0, interest rate r of 0.05, maturity T of 10, volatility σ of 0.25 and the debt level X of 0.45. Notice that $\alpha = 0.5$ and $\alpha = 1$ correspond to $H = 0.225$ and $H = 0.45$ respectively as $H = \alpha X$. Figure 5.1 shows a flat slope for H ranging from 0 to 0.45. The option price keeps almost constant of 0.7 for H in this range. Similarly, Figure 5.2 shows the sensitivity of the DOC option delta to the barrier level H. A flat slope is also observed for H being less than the debt of 0.45. As mentioned in the previous subsection, the ML framework is performed by searching H, V and α such that (5.5) is maximized. Since the DOC option price and DOC delta need to be respected during the maximization process, when both terms are insensitive to the position of barrier H, it would be reasonable that an inaccurate α is estimated. However, the estimates for firm asset values and asset volatility becomes robust to the barrier level that is less than the face value of the liability.

Chapter 6

Estimation with Empirical Data

6.1 Description of data

This chapter presents an empirical investigation over a large cross-section of industrial firms. We collect data from Compustat as well as Datastream. Attention is paid to industrial firms with December fiscal year-ends and SIC codes between 2,000 and 5,999. The sample covers a ten-year period of *daily observations* from 1993 to 2002. The whole data set consists of 13,317 firm-years which provides abundant data in various industrial sectors to perform the maximum likelihood estimation.

Table 6.1 presents some basic statistics of our diverse firms' data. We report the minimums, medians, maximums, means and standard derivations of the debt to equity ratio of 13,317 available firm-years according to industrial sectors. The debt to equity ratio is various as the minimum and the maximum ratios vary quite a lot in different sectors. It can range from around 0.001 to over 5000. This implies that the sample for our empirical analysis takes a large number of firms with various financial structures into account. When all observations are pooled together, it is found that the averaged face value of debts of a firm is about 2.6 times to its market equity value. However, the typical firm has a much smaller debt to equity ratio of about 0.5. The annualized risk-free rate ranges from a low of 1.32% to a high of 5.98% over the sample period. The mean value is about 4.3%.

Table 6.1
Descriptive statistics for the sample

Panel A: Debt to equity ratio

Industrial sector	Number of firm years	Minimum	Median	Maximum	Mean	Standard Derivation
1. Food and beverages (20)	382	0.0336	0.4306	29.3338	1.1030	2.8101
2. Miscellaneous (21,24,25,27,30,31,46,48)	1557	0.0066	0.6865	644.7465	2.3708	17.6009
3. Textile and apparel (22,23)	231	0.0200	0.7526	1067.2545	10.1094	80.4994
4. Paper products(26)	320	0.0091	1.0545	17.0665	1.6308	1.9783
5. Chemicals (28)	1948	0.0016	0.1689	252.5202	1.5408	12.1809
6. Petroleum (29)	149	0.0515	0.6893	9.7840	1.0849	1.1989
7. Stone, clay and glass (32)	139	0.0088	0.7941	143.3097	3.0990	13.9116
8. Primary metals (33)	332	0.0274	1.0348	196.6046	3.4254	12.1530
9. Fabricated metals (34)	358	0.0604	0.6718	97.4935	1.4833	5.4988
10. Machinery (35)	1260	0.0030	0.3479	18.8429	0.6998	1.1738
11. Appliances, electrical equipment (36)	1449	0.0025	0.2786	5554.7237	9.4925	177.5735
12. Transportation equipment (37)	422	0.0117	0.8551	173.2681	2.6338	12.1227
13. Miscellaneous manufacturing (38,39)	1598	0.0015	0.2049	95.2583	0.6857	3.0561
14. Railroads (40)	96	0.1469	1.2535	6.4642	1.5994	1.1533
15. Other transportation (41,42,44,45,47)	542	0.0230	1.3396	59.7711	2.9580	5.9337
16. Utilities (49)	971	0.0199	1.4568	114.6390	2.2389	5.8396
17. Other retail trade (50-52, 54-59)	1536	0.0031	0.6544	49.9581	1.5456	3.0665
18. Department Stores(53)	27	0.0616	1.6225	4.8043	1.5707	1.4832
pooled result	13317	0.0015	0.5151	5554.7237	2.6600	60.1983

Panel B: Risk free rate

Mean	Standard Derivation	Minimum	Median	Maximum
0.0428	0.0148	0.0132	0.0486	0.0598

To estimate barrier-to-debt ratio (α), market value of assets (V) and firm assets volatility (σ), the ML approach requires: the market value of equity (V_E) directly obtained from Compustat and Datastream, the future promised payment (X) measured as the book value of assets minus the book value of equity, the risk free interest rate (r) given by the rate of return of one-year US Treasury bills, and the time to maturity of the option (T). We take a proxy of 10 years for T which has been used in many empirical studies such as Brockman and Turtle (2003).

With a complete specification of V_E, X, r and T, the analysis can be proceeded to the processing stage. The time series of V_E together with the parameter values are substituted into the log-likelihood function of (5.5). Then, it is maximized via the numerical scheme of Nelder-Mead (1965) built-in the software of MATLAB. A convergence analysis for the scheme is reported in Lagarias *et al.*(1998).

6.2 Empirical results

In Chapter 4, we shows theoretically that the taking of the proxy would generate a barrier level (H) over the firm's total liability (X), i.e. $\alpha > 1$. A forceful evidence to see the problem of the proxy is the level of α estimated from the empirical data. The Panel A of Table 6.2 reports that the average α from all available observations is 0.4395 with standard derivation of 1.1989 while the median is 0.0771. It means that the majority of firms have a financial structure of the default barrier level under the total liability. In order to achieve a better understanding of our analysis, we disaggregate the sample by year (Panel B) and industry (Panel C).

Table 6.2

Barrier to debt ratio (α) estimates averaged by year and industrial sectors

	Number of firm years	mean	Standard Derivation	Minimum	Median	Maximum
Panel A: Pool sample results						
pooled	13317	0.4395	1.1989	0	0.0771	36.7619
Panel B: Alpha estimates by year						
1993	888	0.3977	1.0163	0	0.0856	16.7143
1994	972	0.3914	0.9857	0	0.0809	16.7143
1995	1048	0.3940	0.9616	0	0.0859	16.7143
1996	1181	0.4122	1.0091	0	0.0843	16.7143
1997	1288	0.4075	0.9870	0	0.0785	16.7143
1998	1366	0.4123	1.0304	0	0.0769	16.7143
1999	1517	0.4197	1.0903	0	0.0736	16.7143
2000	1668	0.4560	1.1819	0	0.0725	16.7143
2001	1722	0.4793	1.4529	0	0.0723	36.7619
2002	1667	0.5447	1.6637	0	0.0764	36.7619

	Number of firm	mean	Standard Derivation	Minimum	Median	Maximum
Panel C: Alpha estimates by sectors						
1. Food and beverages (20)	48	0.4701	1.2609	0	0.0701	7.9345
2. Miscellaneous (21,24,25,27,30,31,46,48)	231	0.3592	1.0153	0	0.0493	12.5686
3. Textile and apparel (22,23)	29	0.3538	0.4242	0	0.2183	1.6070
4. Paper products(26)	38	0.3782	0.6257	0	0.0899	3.1943
5. Chemicals (28)	273	0.9073	1.9387	0	0.2073	16.7143
6. Petroleum (29)	17	0.1997	0.3384	0	0.0768	1.3102
7. Stone, clay and glass (32)	16	0.6542	1.0352	0	0.3131	3.8239
8. Primary metals (33)	40	0.5871	2.1144	0	0.0425	12.7878
9. Fabricated metals (34)	43	0.3181	0.6588	0	0.0957	3.7817
10. Machinery (35)	164	0.5111	1.4439	0	0.1140	15.1958
11. Appliances, electrical equipment (36)	208	0.5501	2.6002	0	0.0734	36.7619
12. Transportation equipment (37)	51	0.2085	0.4267	0	0.0156	2.0997
13. Miscellaneous manufacturing (38,39)	221	0.9223	2.1553	0	0.2735	26.3167
14. Railroads (40)	11	0.4357	0.5157	0	0.2161	1.3728
15. Other transportation (41,42,44,45,47)	72	0.2686	0.6037	0	0.0123	3.6635
16. Utilities (49)	112	0.1624	0.2723	0	0.0483	1.5118
17. Other retail trade (50-52, 54-59)	214	0.3934	1.3699	0	0.0166	15.3027
18. Department Stores(53)	4	0.2961	0.5878	0	0.0033	1.1778

The α estimates are based on the whole sample of firms assuming an option life of 10 years and 0 rebate level. All other inputs are based on firm-specific data. Industry classification is reported according to two-digit SIC codes in parentheses. The means, standard derivations, minimums, medians and maximums of α are tabulated in corresponding columns.

Panel B presents the α value over the 10-year period of the sample. Both the means and medians of α in all years are less than one but with various values. This implies that the barrier is lower than the debt payment for more than 50% of our sample firms in different economic situations. Panel C reveals that the message of α less than unity is consistent through out different industries. The means and medians of α in all sectors are less than one. However, when we observe the α values in terms of maximum values, some firms really have α larger than one. This suggests that the existence and the level of barrier is very firm specific.

Even though the α estimates in the majority of firms are consistently less than one and small in values, it does not mean that the default barrier is not important in bankruptcy analysis. In fact, the ML approach may underestimate the default barrier by 20% for $\alpha \leq 1$ but gives a good estimates for $\alpha > 1$, as what we have demonstrated in the simulation. Therefore, the empirical study so far only shows that default barriers for the majority of firms are less than the future promised payment. Another factor affecting the estimation quality concerns with the stockholders behavior. For those firms having default barrier under the face value of the debt, the equity price is insensitive to the barrier level. It follows that stockholders may ignore the default barrier when making their investment decisions. This makes the market value of equities is unable to reflect the impact of the default barrier.

Although we realize the potential importance of default barriers, it is difficult for common rational investors to assure the barrier level through the market values of equities and the accounting information. We think that the correct positioning of the default barrier may be more relevant to other information such as bond covenants and bankruptcy rules of the economy. Alternatively, sophisticated statistical estimation can be constructed to enhance the estimation quality. We leave this for future research.

The advantage of the proposed ML estimation approach is that the firm asset value and asset volatility are estimated with a high quality even under the existence of default barrier. It enables us to compare and justify the quality of the proxy firm asset value. To make comparison, we use the following measurement:

$$\begin{array}{c}\text{Percentage difference of}\\ \text{firm asset values}\\ \text{for a firm year}\end{array} = \frac{1}{N_y}\sum_{i=1}^{N_y}\left|\frac{V^i_{proxy} - V^i_{ML}}{V^i_{ML}}\right| \tag{6.1}$$

where N_y is the number of trading days in a particular year, V^i_{proxy} is obtained through the proxy for the ith observation and V^i_{ML} is the firm value estimated from the ith observation through our ML approach. The averaged percentage differences in Table 6.3 are consistently over 28% for all industrial sectors while the medians range from 8% to 50%. For some particular industrial sectors, such as Textile and apparel, the average difference is as large as 77 times. In our sample, we find that high percentage differences happen to highly leveraged companies. To understand this, let us consider the following illustrative example. Suppose a firm has 100 units of market value of equity and 1900 units of liabilities. The firm asset value would be 2000 units if the proxy is used. However, due to the high proportion of debt, the firm is experiencing a high credit risk so that the market of value of corporate debt may be discounted and shrunk a lot. As a consequence, the market value of debt may be diminished to, say 300 units, and the firm asset value collapses to 400 units altogether. This ultimately induces 400 % error in the firm asset value. The implication of this phenomenon is that firm asset values are effectively overstated by employing the proxy. Sometimes, the overestimation can excess 100%.

Table 6.3
The percentage difference between the firm asset value proxy used in Brockman and Turtle (2003) and that estimated by ML method

	number of firm year	mean	Standard Derivation	min	median	max
1. Food and beverages (20)	382	1.0960	4.4092	0.0105	0.2378	30.3125
2. Miscellaneous (21,24,25,27,30,31,46,48)	1557	1.7351	5.8568	0.0031	0.3334	72.7065
3. Textile and apparel (22,23)	231	77.3441	378.4225	0.0406	0.3240	2036.8846
4. Paper products(26)	320	0.6675	1.4118	0.0318	0.2665	7.7745
5. Chemicals (28)	1948	0.5245	3.2155	0.0025	0.0831	41.4918
6. Petroleum (29)	149	0.3858	0.3362	0.1121	0.2440	1.1370
7. Stone, clay and glass (32)	139	0.6653	0.7688	0.0234	0.2300	2.2380
8. Primary metals (33)	332	0.8915	1.1701	0.0659	0.3347	4.2128
9. Fabricated metals (34)	358	1.0470	4.1396	0.0290	0.2138	27.3769
10. Machinery (35)	1260	0.2870	0.3548	0.0120	0.1858	2.4490
11. Appliances, electrical equipment (36)	1449	10.8753	150.8159	0.0044	0.1472	2175.3811
12. Transportation equipment (37)	422	4.0073	22.5143	0.0070	0.3466	161.2784
13. Miscellaneous manufacturing (38,39)	1598	0.4137	1.7565	0.0029	0.0942	18.3683
14. Railroads (40)	96	0.5652	0.6603	0.0103	0.2922	2.1144
15. Other transportation (41,42,44,45,47)	542	1.0662	1.6414	0.0229	0.5020	11.0618
16. Utilities (49)	971	0.9833	5.5087	0.0320	0.3338	58.5552
17. Other retail trade (50-52, 54-59)	1536	1.0854	2.4106	0.0040	0.3408	16.3799
18. Department Stores(53)	27	0.4732	0.5156	0.0240	0.4344	0.9998

The last estimating parameter is the asset value volatility. In Table 6.4, the percentage difference between the firm asset volatility estimated from the use of the proxy and that estimated by the ML method has been calculated. σ_{proxy} denotes the annualized standard derivation for the proxy asset values of a firm over the time horizon of all available equity data. By the same token, we use the measurement:

$$\begin{array}{c}\text{Percentage difference of}\\ \text{firm asset volatility}\\ \text{for a firm}\end{array} = \left|\frac{\sigma_{proxy} - \sigma_{ML}}{\sigma_{ML}}\right|. \tag{6.2}$$

Means and medians of the percentage difference of firm asset volatility for all industrial sectors are all over 10%. As the ML method provides a good estimate of firm asset volatility which has been shown in Table 5.1, the percentage difference indicates that the volatility used in DOC framework would be inappropriate when the proxy is considered.

Table 6.4

The percentage difference between the firm asset volatility in Brockman and Turtle (2003) and that estimated by ML method

	Number of firms	Mean	Standard Derivation	Minimum	Median	Maximum
1. Food and beverages (20)	48	0.2362	0.1983	0.0110	0.1937	0.9525
2. Miscellaneous (21,24,25,27,30,31,46,48)	231	0.3643	0.9370	0.0069	0.2397	13.9022
3. Textile and apparel (22,23)	29	0.3109	0.2763	0.0292	0.2364	0.9958
4. Paper products(26)	38	0.2658	0.2811	0.0083	0.2033	1.5904
5. Chemicals (28)	273	0.1350	0.1580	0.0006	0.0728	0.8962
6. Petroleum (29)	17	0.2274	0.0900	0.0960	0.2131	0.3481
7. Stone, clay and glass (32)	16	0.3011	0.1836	0.0423	0.2540	0.6394
8. Primary metals (33)	40	0.2843	0.1362	0.0292	0.2708	0.6639
9. Fabricated metals (34)	43	0.2247	0.1781	0.0078	0.1842	0.8245
10. Machinery (35)	164	0.1731	0.1297	0.0002	0.1532	0.6423
11. Appliances, electrical equipment (36)	208	0.1601	0.1494	0.0005	0.1105	0.9307
12. Transportation equipment (37)	51	0.2937	0.2256	0.0084	0.2053	0.9225
13. Miscellaneous manufacturing (38,39)	221	0.1498	0.1547	0.0000	0.0897	0.8330
14. Railroads (40)	11	0.2723	0.1780	0.0052	0.2659	0.6409
15. Other transportation (41,42,44,45,47)	72	0.3702	0.2132	0.0149	0.3299	0.8199
16. Utilities (49)	112	0.3384	0.6172	0.0051	0.2498	6.5125
17. Other retail trade (50-52, 54-59)	214	0.3187	0.5136	0.0038	0.2206	7.0503
18. Department Stores(53)	4	0.2062	0.2010	0.0421	0.1646	0.4533

Chapter 7

Conclusion

This thesis provides a strong theoretical support to show that using the sum of the market value of equity and the book value of corporate liabilities as a proxy for the market value of the firm's assets is incorrect in validating the existence of default barrier under the barrier option pricing framework. We apply options' properties to prove that adopting the proxy would generate a positive default barrier regardless of what market data observed. In order to recognize the default barriers from the market information, a maximum likelihood estimation approach is proposed to estimate the barrier, the market value of firm's asset and asset value volatility with a simulation-based verification for its performance. The proposed framework is then applied to an empirical study.

The empirical study shows that the default barriers of majority of firms in our sample are located under the book value of the corporate liabilities. The position of the barrier level may be underestimated because of insensitivity of DOC option and DOC delta to the barrier level H when α is less than 1. But, the performance of the estimates of the firm asset value and asset volatility remain very accurate disrespect the value of α. This renders us to compare the difference between the estimates from the proxy and that from the maximum likelihood approach. The firm asset values can be overstated by 28% or even more than 100% if the proxy is employed to estimate the firm asset values. The asset volatility implied by the proxy firm values is also overestimated in comparing with that obtained from the maximum likelihood approach.

From the theoretical arguments of this thesis, it teaches us a lesson that we should have awareness when using proxy especially in estimating structural models. We should be very careful for taking any proxy so as to ensure that the proxy would not induce any contradictory situation. Moreover, from the empirical study, default barriers of firms are various even if they are in the same industrial sector. This implies firms in the same industry can share dissimilar creditworthiness. Hence, default barriers should be estimated in a firm-specific manner.

In order to enhance the performance of the present approach, a default database can be included in our analysis since the default positions of firms are more informative to achieve a better estimation of the barrier level. Also, sophisticated statistical methods such as Bayesian framework can be applied to improve the estimation quality. We leave this for future research.

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